



FUKA

A new public code for initial data of unequal-mass,
spinning compact-object binaries

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<https://kadath.obspm.fr/fuka/>

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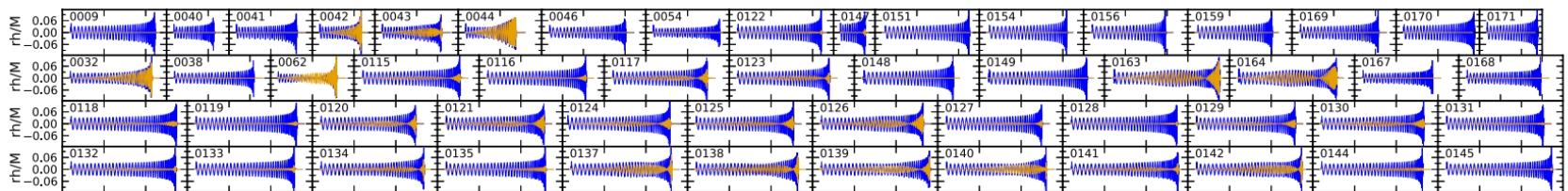
Outline

- 1 Motivation
- 2 Analytical Framework
- 3 Numerical Implementation
- 4 Capability Details
- 5 Results
- 6 Demonstration
- 7 Conclusion

Motivation

The primary motivation of this code base is to provide consistent initial data solutions to Einstein's field equations

- ① Gravitation wave extraction
- ② Study of merger dynamics in numerical simulations with configurations such as
 - Binary black holes (BBH) with high mass ratio with mixed spins,
 - Binary neutron stars (BNS) with high mass ratio and mixed spins,
 - Black hole - neutron stars (BHNS) with mass ratio and mixed spins,



<https://link.aps.org/doi/10.1103/PhysRevLett.111.241104>

XCTS Constraint Equations

By choosing $\partial_t \tilde{\gamma}_{ij} = \partial_t K = K = 0$, where $\tilde{\gamma}_{ij}$ is the flat metric, the XCTS formulation results in the following set of coupled elliptic differential equations¹

$$\tilde{D}^2 \Psi = -\frac{1}{8} \Psi^{-7} \hat{A}_{ij} \hat{A}^{ij} \underbrace{-2\pi \Psi^5 E}$$

$$\tilde{D}^2 (\alpha \Psi) = \frac{7}{8} \alpha \Psi^{-7} \hat{A}_{ij} \hat{A}^{ij} \underbrace{+2\pi \alpha \Psi^5 (\tilde{E} + 2\tilde{S})}$$

$$\tilde{D}^2 \beta^i = -\frac{1}{3} \tilde{D}^i \tilde{D}_j \beta^j + 2\hat{A}^{ij} \tilde{D}_j (\alpha \Psi^{-6}) \underbrace{+16\pi \alpha \Psi^4 \tilde{j}^i}$$

$$\hat{A}^{ij} \equiv \frac{\Psi^6}{2\alpha} (\tilde{D}^i \beta^j + \tilde{D}^j \beta^i - \frac{2}{3} \tilde{\gamma}^{ij} \tilde{D}_k \beta^k).$$

Where the source terms are defined as:

$$E := \rho h W^2 - p$$

$$S^i := 3p + (E + p)U^2$$

$$j^i := \rho h W^2 U^i$$

¹PhysRevD.90.064006, eq.7-10

XCTS Constraint Equations

By choosing $\partial_t \tilde{\gamma}_{ij} = \partial_t K = K = 0$, where $\tilde{\gamma}_{ij}$ is the flat metric, the XCTS formulation results in the following set of coupled elliptic differential equations

$$\frac{p}{\rho} \tilde{D}^2 \Psi = \frac{p}{\rho} \left[-\frac{1}{8} \Psi^{-7} \hat{A}_{ij} \hat{A}^{ij} - 2\pi \Psi^5 E \right]$$

$$\frac{p}{\rho} \tilde{D}^2 (\alpha \Psi) = \frac{p}{\rho} \left[\frac{7}{8} \alpha \Psi^{-7} \hat{A}_{ij} \hat{A}^{ij} + 2\pi \alpha \Psi^5 (\tilde{E} + 2\tilde{S}) \right]$$

$$\frac{p}{\rho} \tilde{D}^2 \beta^i = \frac{p}{\rho} \left[-\frac{1}{3} \tilde{D}^i \tilde{D}_j \beta^j + 2\hat{A}^{ij} \tilde{D}_j (\alpha \Psi^{-6}) + 16\pi \alpha \Psi^4 j^i \right]$$

$$\hat{A}^{ij} \equiv \frac{\Psi^6}{2\alpha} (\tilde{D}^i \beta^j + \tilde{D}^j \beta^i - \frac{2}{3} \tilde{\gamma}^{ij} \tilde{D}_k \beta^k).$$

Where the source terms are defined as:

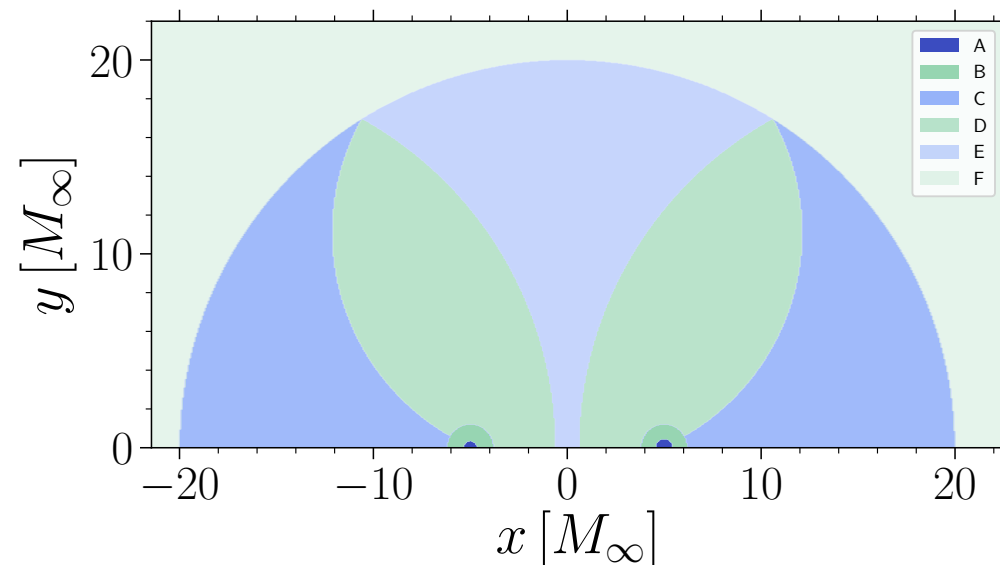
$$E := \rho h W^2 - p$$

$$S^i := 3p + (E + p)U^2$$

$$j^i := \rho h W^2 U^i$$

Implementation

- FUKA is based on the Kadath² spectral solver library
 - Equations can be implemented in a \LaTeX -like syntax
 - New operators can be added modularly
- ID involving Black Holes are constructed using excision conditions and quasi-equilibrium ($M_K - M_{\text{ADM}} = 0$)
- ID involving Neutron stars can be constructed using piecewise polytrope or tabulated Equations of State



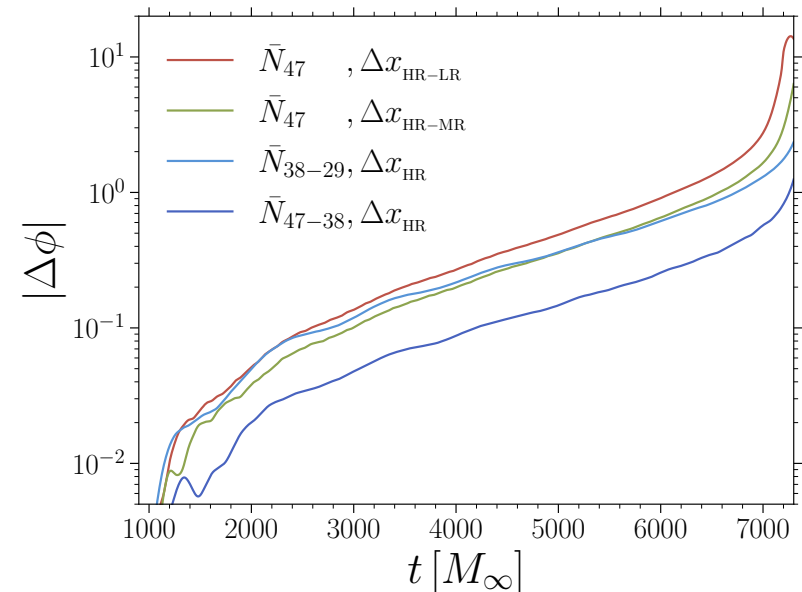
²<https://kadath.obspm.fr/>

Capabilities Map

- ✓ Isolated black hole with arbitrary mass and spin
- ✓ Isolated neutron star with arbitrary mass and spin
- ✓ Binary black holes with high mass ratio with mixed spins
- ✓ Binary neutron stars with high mass ratio and mixed spins
- ✓ Black Hole - Neutron stars with mass ratio and mixed spins
- ✓ Maximal spins (anti-)aligned with orbital axis using conformal flatness approximation
- ✓ Eccentricity reduced binaries
- ✓ Minimize ADM linear momentums
- ✓ Provide an extensible framework for continued development
- ✗ Provide export interfaces to streamline importing into evolution environments

- ① The error contribution from the ID solution (e.g. $H|_{t=0} \approx 0$)
- ② The error of the evolution scheme
 - Analytical formulation and numerical implementation used
 - Finite resolution of the evolution
- ③ Note: $\tilde{N} = N^{1/3}$ where N is the total number of colocation points

Error Sources



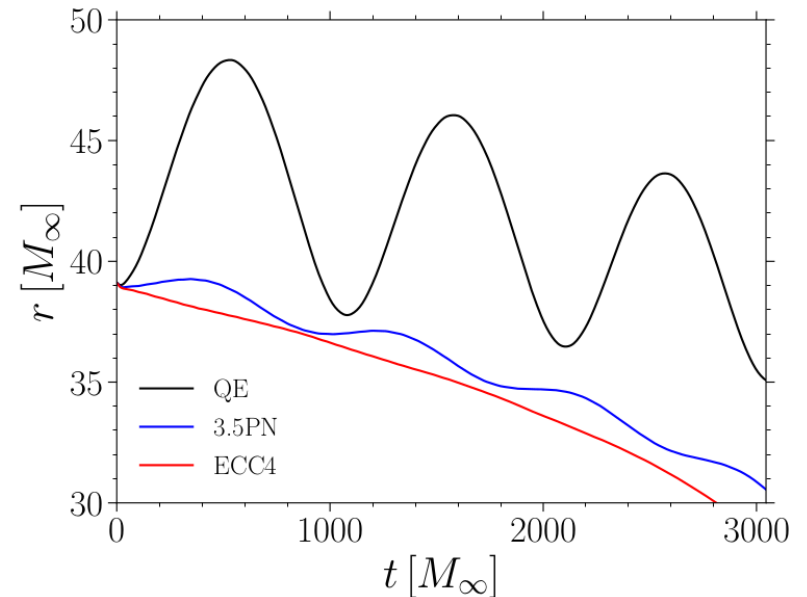
$$M_{\text{b}_{<1/2>}} = 1.4946, q = 1$$

$$\chi_1 = 0, \chi_2 = 0$$

$$\text{EOS} = \text{SLy}$$

BNS - Eccentricity Comparison

- QE is necessary starting assumption, but a poor approximation
- 3.5 PN already provides a drastic improvement
- Further improvements can be made using iterative eccentricity reduction



$$M_\infty = 2.7, q = 0.6875$$

$$\chi_1 = 0, \chi_2 = 0.6$$

$$\text{EOS} = \text{TNTYST}$$

BHNS - Eccentricity Reduction

Example

$$M_{BH} = 2.42M_{\odot}, \chi_{BH} = 0.52 \quad M_{NS} = 1.17M_{\odot}, \chi_{NS} = 0.6$$
$$\Delta x_{min} = 0.2, \eta = 0.5, \kappa = 0.02$$

Solver Layout

- Each solver includes the following:
 - setup: setup numerical space and initialize fields
 - solve: solve the input dataset
 - increase_resolution: restructure the numerical space
 - reader: useful diagnostic code
- Reusing ID is essential
 - FUKA is computationally demanding
 - On 128 cores:
 - Isolated objects take 2-4min for low resolution
 - Binaries take 1-2hrs
- High resolution is expensive, but can be minimized using resolution increase capabilities

Demo

```

codes/NS:~>./bin/Release/setup
Using default Togashi setup.
*****ns info*****
                chi: 0
                dim: 3
                hc: 1.2591
                madm: 1.4
                mb: 1.55
                nc: 0.00137
nshells: 0
omega: 0
ql_madm: 1.4
qpig: 12.5664
res: 9
rin: 3.1
rmid: 6.2
rout: 9.3
eosfile: togashi.lorene
eostype: Cold_Table
h_cut: 0
interpolation_pts: 2000

```

```

codes/NS:~>ls -lt
initns.dat
initns.info
sfho_pwp
togashi
build
bin
src
README
CMakeLists.txt
bench
compile

```

- Running "setup" will generate the default dataset: initns.dat/info
- DAT file contains numerical space and scalar fields
- INFO file contains scalar quantities and drives the respective codes

Demo

- The initial section can maintain a history of iterations
- The NS section describes numerical grid setup and related scalar quantities

```

1 initial
2 {
3     chi 0
4     dim 3
5     hc 1.0094227447624744
6     madm 1.3999999999999999
7     mb 1.55
8     nc 0.0013699999999999999
9     nshells 0
10    omega 0
11    ql_madm 1.3999999999999999
12    qpig 12.566370614359172
13    res 9
14    rin 3.0341687888025062
15    rmid 6.0683375776050124
16    rout 9.1025063664075176
17    eosfile togashi.lorene
18    eostype Cold_Table
19    h_cut 0
20    interpolation_pts 2000
21 }
22 ns
23 {
24     chi 0
25     dim 3
26     hc 1.0094227447624744
27     madm 1.3999999999999999
28     mb 1.55
29     nc 0.0013699999999999999
30     nshells 0
31     omega 0
32     ql_madm 1.3999999999999999
33     qpig 12.566370614359172
34     res 9
35     rin 3.0341687888025062
36     rmid 6.0683375776050124
37     rout 9.1025063664075176
38     eosfile togashi.lorene
39     eostype Cold_Table
40     h_cut 0
41     interpolation_pts 2000
42 }

```

Demo

The fixing parameters consist of:

- "chi", dimensionless spin parameter (χ)
- "madm", Total ADM mass of the isolated NS
- "eosfile", the file that describes the tabulated or polytropic EOS
- "eostype", Cold_PWPoly or Cold_Table

```

22 ns
23 {
24   chi 0
25   dim 3
26   hc 1.0094227447624744
27   madm 1.3999999999999999
28   mb 1.55
29   nc 0.0013699999999999999
30   nshells 0
31   omega 0
32   ql_madm 1.3999999999999999
33   qpig 12.566370614359172
34   res 9
35   rin 3.0341687888025062
36   rmid 6.0683375776050124
37   rout 9.1025063664075176
38   eosfile togashi.lorene
39   eostype Cold_Table
40   h_cut 0
41   interpolation_pts 2000
42 }
```

Demo

```
=====  
Iter:      +2  
Error:     +0.00154638  
Mb:        +1.62137  
Madm:      +1.4  
Madm_ql:   +1.4 [+3.93971e-13]  
Mk:        +1.20369 [+0.140223]  
R:         +6.33379 +6.33379
```

```
Error init = 4.23875e-05, Eq: firstint Dom: 0 Bounday: -1  
DOF / rank = 6598 x 6598 / 128 = 340106  
Block size is 32, consider lowering the number of cores.  
Load and redistrib Jacobian: 7.0127 / 7.0061 / 7.01712 seconds (avg/min/max)  
Inverting the matrix: 0.366115 seconds  
Distributing solution: 0.0143809 seconds
```

```
codes/NS:~>ls -l togashi/  
converged_NS.1.4.0.09.dat  
converged_NS.1.4.0.09.info  
converged_NS_norot_bc.1.4.0.09.dat  
converged_NS_norot_bc.1.4.0.09.info  
codes/NS:~>cp togashi/converged_NS.1.4.0.09.info  initns.info  
codes/NS:~>cp togashi/converged_NS.1.4.0.09.dat  initns.dat  
codes/NS:~>vim initns.info
```

```

22 ns
23 {
24   chi 0.3
25   dim 3
26   hc 1.260774576466819
27   madm 1.5
28   mb 1.5525575066925488
29   nc 0.0013740471269387219
30   nshells 0
31   omega 0
32   ql_madm 1.3999999999999999
33   qpig 12.566370614359172
34   res 9
35   rin 3.0341687888025062
36   rmid 6.0683375776050124
37   rout 9.1025063664075176
38   eosfile togashi.lorene
39   eostype Cold_Table
40   h_cut 0
41   interpolation_pts 2000
42 }

Last Stage Enabled: total_bc
=====
Single star solver 3D
*****ns info*****
                chi: 0.3
                dim: 3
                hc: 1.26077
                madm: 1.5
                mb: 1.55256
                nc: 0.00137405
                nshells: 0
                omega: 0
                ql_madm: 1.4
                qpig: 12.5664
                res: 9
                rin: 3.03417
                rmid: 6.06834
                rout: 9.10251
                eosfile: togashi.lorene
                eostype: Cold_Table
                h_cut: 0
                interpolation_pts: 2000
=====
#####
Rotating - with fixed MADM
#####
Error init = 0.675, Eq: integ(intJ) - chi * Madm * Madm = 0 Dom: 3 Bounday: 1
DOF / rank = 12853 x 12853 / 128 = 1.29062e+06
Load and redistrib Jacobian: 44.4222 / 44.3911 / 44.4487 seconds (avg/min/max)
Inverting the matrix: 1.27914 seconds
Distributing solution: 0.0209185 seconds
=====

```



```
22 ns
23 {
24     chi 0
25     dim 3
26     hc 1.0094227447624744
27     madm 1.3999999999999999
28     mb 1.55
29     nc 0.0013699999999999999
30     nshells 0
31     omega 0
32     ql_madm 1.3999999999999999
33     qpig 12.566370614359172
34     res 9
35     rin 3.0341687888025062
36     rmid 6.0683375776050124
37     rout 9.1025063664075176
38     eosfile togashi.lorene
39     eostype Cold_Table
40     h_cut 0
41     interpolation_pts 2000
42 }
```

```
kadath/eos:->ls -l
apr4.polytrope
APR+VQCD_simple.lorene
APR+VQCD_simple_no_phep.lorene
bhblp.lorene
dd2.lorene
gam2.polytrope
poly2_123.6.lorene
poly2.lorene
sfho.lorene
sly4.polytrope
sly_pwp.lorene
togashi.lorene
kadath/eos:->
```

```

22 ns
23 {
24     chi 0
25     dim 3
26     hc 1.0094227447624744
27     madm 1.3999999999999999
28     mb 1.55
29     nc 0.0013699999999999999
30     nshells 0
31     omega 0
32     ql_madm 1.3999999999999999
33     qpig 12.566370614359172
34     res 9
35     rin 3.0341687888025062
36     rmid 6.0683375776050124
37     rout 9.1025063664075176
38     eosfile sly4.polytrope
39     eostype Cold_PWPoly
40     h_cut 0
41     interpolation_pts 2000
42 }

codes/NS: ->mkdir sfho_pwp
codes/NS: ->./bin/Release/setup initns.info
Creating setup from: initns.info
*****ns info*****
                chi: 0
                dim: 3
                hc: 1.23905
                madm: 1.4
                mb: 1.55
                nc: 0.00137
                nshells: 0
                omega: 0
                ql_madm: 1.4
                qpig: 12.5664
                res: 9
                rin: 3.03417
                rmid: 6.06834
                rout: 9.10251
                eosfile: sly4.polytrope
                eostype: Cold_PWPoly
                h_cut: 0
                interpolation_pts: 2000

codes/NS: ->
    
```

Demo

```
codes/NS:->./bin/Release/increase_resolution sfho_pwp/converged_NS.1.4.0.09.info initns 11
Resolution of old space: 9 (r), 9 (theta), 8 (phi)
Rmin/max: 6.28166 6.28166
Resolution of new space: 11 (r), 11 (theta), 10 (phi)
Bounds: Array of 1 dimension(s)
3.14083 6.28166 9.42249
```

```
*****ns info*****
      chi: 0
      dim: 3
      hc: 1.2594
      madm: 1.4
      mb: 1.5593
      nc: 0.00143
      nshells: 0
      omega: 0
      ql_madm: 1.4
      qpig: 12.5664
      res: 11
      rin: 3.14083
      rmid: 6.28166
      rout: 9.42249
      eosfile: sly4.polytrope
      eostype: Cold_PWPoly
      h_cut: 0
      interpolation_pts: 2000
```

Demo

```
codes/BNS:->./bin/Release/setup ../NS/sfho_pwp/converged_NS.1.4.0.09.info 50
```

```
*****binary info*****
      com: 0
      distance: 33.8524
      global_omega: 0.007676
      outer_shells: 0
           q: 1
      qpig: 12.5664
      res: 9
      rext: 67.7048
*****ns info*****
      chi: 0
      dim: 3
      hc: 1.2594
      madm: 1.4
      mb: 1.5593
      nc: 0.00143
      nshells: 0
      omega: 0
      ql_madm: 1.4
      qpig: 12.5664
      res: 9
      rin: 3.14083
      rmid: 6.28166
      rout: 9.42249
      eosfile: sly4.polytrope
      eostype: Cold_PWPoly
      h_cut: 0
      interpolation_pts: 2000
*****ns info*****
      chi: 0
      dim: 3
      hc: 1.2594
      madm: 1.4
      mb: 1.5593
      nc: 0.00143
      nshells: 0
      omega: 0
      ql_madm: 1.4
      qpig: 12.5664
      res: 9
      rin: 3.14083
      rmid: 6.28166
      rout: 9.42249

Rmin/max:
6.28166 6.28166
Bounds:
NS1: 3.14083 6.28166 9.42249
NS2: 3.14083 6.28166 9.42249
```

- BNS ID is initialized from an isolated TOV solution resulting in initbin.info/dat
- It's best to solve using an irrotational setup first before making changes to spin and mass ratio
- See the README for more details

Summary

- FUKA is a collection of ID solvers which provide consistent solutions to Einstein's field equations
- Binary solvers provide an easy interface to perform eccentricity reduction using PN estimates or iterative corrections
- Computational requirements are minimized by reusing previous ID for iterative solutions and by running at high resolution only when needed
- Internal exporters provide an easy interface to evolution codes
 - ETK thorns are available at: <https://bitbucket.org/fukaws/>
- How can FUKA ID help your research?
 - Study eccentricity reduced inspirals using iterative reduction or PN estimates
 - Exploration of merger dynamics of a large spectrum of BBH, BHNS, and BNS configuration

Future Work

- ① Transition to imported boosted objects - if this proves reliable
- ② Solver ver.2 is in the works but will not replace ver.1
- ③ Extend BBH and BHNS codes to include imposing Kerr solution on the horizon to achieve extremal spins ($\chi \approx 0.995$)
- ④ Improve solvers, documentation, and exporters based on community feedback